An Innovative Modeling Course at the Freshman/Sophomore Level

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Abstract

This paper will present an overview of a new modeling course designed with a Course and Curriculum Development grant from the National Science Foundation.* The unique feature of this course is its intended audience: non-science majors, especially students who take only one mathematics course during their undergraduate program. The course is based on applications from across the disciplines and focuses on model development, test of model against data and interpretation of results. Models are both discrete and continuous, with quantitative as well as qualitative interpretations of long-term behavior and sensitivity to parameters. Advanced computations are performed by the computer algebra system Mathematica [1] using an intuitive point-and-click interface and special packages developed by the author. This customized interface frees the student from the usual beginner's frustrations of learning Mathematica, and assists in interactively exploring "What if?" scenarios. Overall, the goal of this course is to make the student a critical consumer of models, to understand what goes into a model as well as its possible limitations.

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1 Introduction

Historically, modeling courses required the calculus sequence and a differential equations course as a prerequisite. This background was necessary to equip students with techniques to derive analytical solutions. Obviously, the target audience of non-science majors will not have this background. However, the advances of computer algebra systems such as Mathematica and Maple [2] have made it possible to create a course for this audience. Model equations can be displayed graphically (for example as direction fields) and qualitative features can be derived from these graphs. "What if?" scenarios can be explored by interactively changing model parameters and seeing the effect on the model outcome. Numerical solutions can be easily performed without knowledge of a programming language.

So why should these non-science majors take a modeling course? Even though they may not need to build models themselves, they are consumers of models. Their lives are affected by policy decisions based on model predictions, for example the issues surrounding global warming. The course will provide these students with an understanding of the modeling process, in particular the fact that models need to be validated by data, and that their predictions may be limited. Students will be able to critically evaluate possibly conflicting information and put it into the proper context.

2 Incorporation of Technology

Use of technology, be it computers or graphing calculators, introduces an additional level of difficulty for students in any course. To minimize the learning curve and to alleviate the usual beginner's frustrations with Mathematica, the author has developed a set of packages with functions specifically designed for the modeling course. To access this rich library of functions with an intuitive interface, the palette feature of Mathematica 3.0. has been utilized. This point-and-click interface hides most of the of the Mathematica programming language, thus combining the power of the computer algebra system with the ease of use of specialized programs such as MacMath [3] for differential equations.

For each major section of the course, a package with functions relating to the particular topic has been developed. These functions are pasted into the notebook by clicking on a button in the corresponding palette, thus avoiding frustrations from typos or incorrect use of parentheses and brackets. All these palettes can be accessed from within any notebook via the File→ Palettes menu. Figure 1 (on the next page) shows the palette for fitting functions to data.

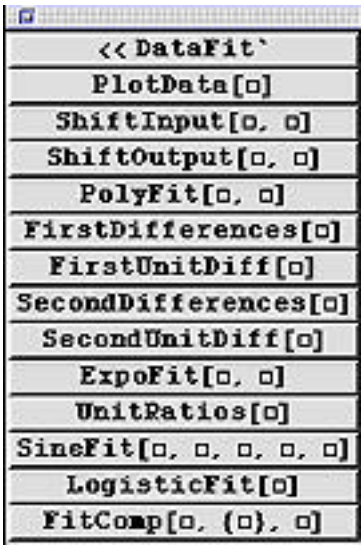


Figure 1: The DataFit palette

In tandem with each palette, a help file explaining the different palette functions is accessible from the File→ Notebooks menu. Each help file gives an introduction on how it is to be used, and has hyper link buttons for all the functions described. Clicking on a button links the student to a full description on how to use the function, followed by a specific example. Figure 2 shows a screen shot of the hyper link portion in the DataFit help file.

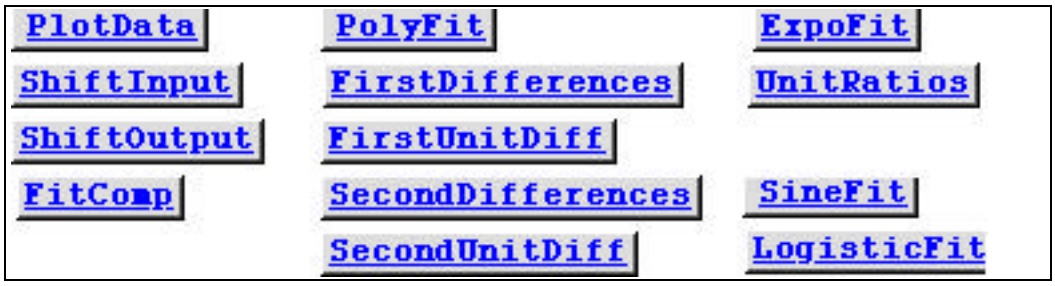


Figure 2: Hyper link buttons in help file

3 Course Content

The course is divided into three principal parts. Part I defines the concept of a function and its various representations (list of values, graph, or formula). Polynomials, exponential, logistic and sine functions are introduced. Students

explore characteristics such as constant first or second differences and features of the shapes of the different types of functions. These properties are then used to decide which type of function should be fitted to given real-world data from sources such as the Statistical Abstracts [4]. The method of least squares is illustrated graphically to give students an understanding on how the fitted function is derived.

Part II covers discrete models. The example of paying off debt on a credit card (see the example in section 5.3 below) is used to define and discuss equilibrium values, stability and long-term behavior in a concrete context. From this example with known underlying dynamics (monthly interest rate and payment amount, the students progress to applications that illustrate the iterative nature of the modeling process: deriving a model, solving it (with the help of Mathematica), comparing it to data, and if necessary to refine it. Matrix notation and arithmetic is defined when generalizing to systems of equations. Related concepts such as eigenvalues and eigenvectors are introduced graphically through an animation. Qualitative and quantitative features of long-term behavior can then be derived from the general solution expressed in terms of the eigenvalues and eigenvectors.

Part III deals with continuous models. To derive the model equation(s), the interpretation of the derivative as rate of change is introduced through the example of average and instantaneous velocity. Side by side with the numerical derivation, the graphical interpretation as limit of secant slopes is developed and visualized through an animation. The two notions of the derivative are connected in the visualization of the differential equation through its slope field. Using the slope field, students will explore qualitative features of the model solution and sensitivity of the long-term behavior to initial conditions.

4 Pedagogy

A major concern in any course that relies to a great amount on technology is the creation of a black-box approach, where the computer is the source of all answers blindly accepted by the students. To avoid this problem, students are performing computations using traditional paper and pencil (whenever suitable) to strengthen their conceptual understanding before using the corresponding palette functions. Furthermore, students are required to make predictions about the model results before solving the model using Mathematica. Model results are discussed and interpreted in the context of the problem both orally and in writing.

Student assignments consist of exercises reinforcing new definitions or methods, as well as open ended questions that require both computations and writing.

These exercises extend models presented in class by either asking additional questions based on the model or by introducing variations to the model. A final class project guides students through the complete modeling process: formulating a model, testing it against data, discussing possible modifications and justifying their final choice for the model. These projects are presented to the class either orally or as a poster session.

5 Implementation

This section presents examples from the course that illustrate the use of the interface developed by the author and the pedagogical approach.

5.1 Functions and their Properties

Functions are introduced as relations between input and output values and their different representations as table of values, graph or formula are illustrated. Specific types of functions are introduced one at a time. Below is an example of an activity for the quadratic function.

Activity Here are a few examples of quadratic functions. For each, make a table of values and draw the points in a coordinate system. Try to find a general description for this family of functions.

a) $f(x) = \frac{1}{3}(x - 2)^2$

b) $g(\mathrm{t}) = -t^2 - 6t - 5$

c) $h(\mathrm{u}) = u^2 - 2$

d) $f(t) = 2t^2 - 8$

Figure 3: Sample activity for quadratic function

After the students have had some experience with (pencil and paper) graphing of the quadratic function and discovering the general form of a quadratic function, the important feature of constant second differences is derived. Students at this point have already computed first differences for linear functions. They compute (by hand) second differences for the functions above. This "hands-on" computation is very important to connect quadratic functions with constant second differences. In addition, it helps students to understand exactly what the palette functions SecondDifferences and SecondUnitDiff compute.

5.2 Fitting Functions to Data

After the properties of the specific types of functions have been established, students will be using Mathematica to do the actual fit. Here is an example of a question in this section. Use of the author defined functions allows the students to concentrate on finding the appropriate type of function, as well as the interpretation of the resulting model, rather than being bogged down in the computation of second differences or unit ratios.

Activity The table below gives the death rate (number of deaths per thousand people) for Americans in different age groups.

Age	Death Rate
40	2.2
45	3.0
50	5.0
55	8.0
60	12.6
65	18.7

a) Graph this data using the function **PlotData**. From the shape of the graph determine which type of function could be fitted. If more than one type is possible, list them all.

b) Use the appropriate functions (such as **FirstDifferences**, S**econdDifferences**, etc.) to validate your guess.

c) Fit the resulting type(s) of function to the data.

d) How good is the fit? If you had several possible candidates for functions, compare them with the function **FitComp**.

e) Does the function type you fitted make sense in the context of the problem? If so, give a reason, if not, argue your case.

f) What does your model predict as the death rate for a 5 year old? ... a 100 year old?

Figure 4: Sample Activity for fitting functions to data

5.3 Discrete Dynamical Systems I (Single Equation)

In the second and third part of the course, models are built from assumptions which translate into model equations. One paradigm that can be used in a wide variety of models is given by

new = old + change.

Below is an example that uses this relation to derive the model.

Activity Suppose you have a credit card with the *Students R Us* bank. When you started college, they gave you a credit card without requiring a long credit history. Unfortunately, you have been using this credit card very heavily over the holidays and you current balance has soared to $5,649. Every time you open a credit card statement, you see your list of purchases followed by an ever-increasing amount of interest charged on the outstanding balance. You started 1998 with the firm commitment to pay off this credit card by paying a specified amount every month and not accruing any new charges. You are wondering how long it will take you to pay off the credit card. When you read the fine print on the statement you learned that the annual percentage rate is 12.9%, and the monthly rate is 1.075%.

a) Let $x(n)$ be the balance (in dollars) n months from now and d the amount you pay off per month. Derive the equation for $x(n+1)$ as a function of $x(n)$.

b) Identify the function f such that $x(n+1) = f(x(n))$.

c) Use **IteratedValue** to compute your balance for the next 12 months if you pay off $25 per month. Do you get any closer to your goal of paying off your debt?

d) Try out what happens if you pay off $50,...$75 per month. Plot the sequences for these three different amounts being paid off per month. What can you say about the results?

e) It seems as if there is a value for d for which the credit card balance stays the same forever. Try out different values for d to determine this value. What happens if you pay less or more than this amount per month?

Figure 5: Example for Discrete Dynamical System I

The DSS1 palette provides functions to compute the system values iteratively. In addition, a graphical representation of the system behavior is provided by the two functions MapIt and LiveIt. Figure 6 shows the output for MapIt for five iterations of the credit card balance in the example above, with $75 being paid off each month. LiveIt provides an animation of the intermediate steps. Both these functions provide an alternative avenue to discuss the stability of the equilibrium.

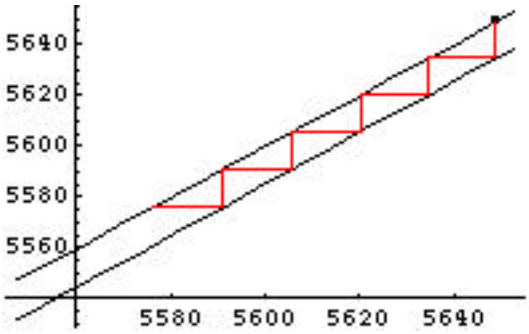


Figure 6: Cobb-web graph for credit card example

5.4 Discrete Dynamical Systems II (Multiple Equations)

This section looks at two or more quantities of interest whose values depend on each other. Examples are predator prey systems and infectious diseases. Starting from the set of equations, matrix notation is introduced as a way to capture the essence of the system. The function InOutVec displays an input vector $\vec{x}$ together with the output vector $A\vec{x}$. Figure 7 shows an example of this function.

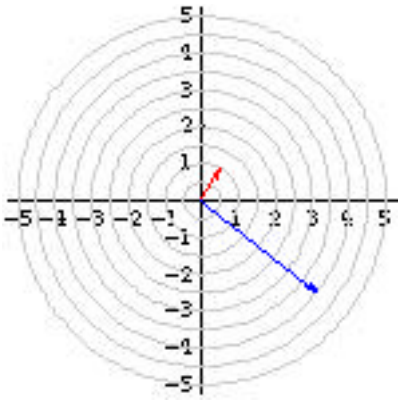


Figure 7: Graphical representation of input and output vector

The function LiveVecs creates an animation of pairs of input/output vectors which include the eigenvectors of the system matrix. This animation allows students to discover the special vectors for which the output vector is a multiple of the input vector. At this point, students compute several iterations of system values and are guided towards the realization that the solution can be expressed in terms of the eigenvectors and eigenvalues. After this illustrative example, students are now using a palette function to display the general solution to the system in terms of the eigenvalues and eigenvectors. From this form, the long-term behavior can be determined.

5.5 Tit-for-Tat

Game theory is used to explain why animals cooperate with one another to their mutual benefit, even though a single animal would benefit even more if it would take advantage of the cooperation of the group. Students will play a sequence of prisoner's dilemma games against the computer. The goal is to find a strategy that results in as high a payoff as possible. The computer employs tit-for-tat, that is it uses the student last move as its next move. This leads into a discussion of the overall performance of a strategy and also to detecting the computer's strategy. It has been shown in round robin tournaments that tit-for-tat is the best possible strategy against opponents with possibly different strategies [5]. Based on this simulation, students explore what happens when animals with different "genetic" strategies interact with each other and which "genetic" strategy will be surviving.

5.6 Internet Resources

The computer lab in which the course is held is connected to the Internet. Students will use web pages for simulations or background information both in class and for assignments. Below is a short selection of web sites of interest, some containing data, others simulations or background information on a specific topic.

- http://www.aqmd.gov:
  The South Coast AQMD is the smog control agency for all or portions of Los Angeles, Orange, Riverside and San Bernardino Counties, California, where 14 million people breathe the dirtiest air in the US.
- http://serendip.brynmawr.edu/~ann/pd.html
  Play a game of Prisoner's Dilemma
- http://www.biozentrum.uni-wuerzburg.de/~brembs/ipd/ipd.html:
  Chaos, cheating and cooperation: potential solutions to the Prisoner's Dilemma

- http://es.rice.edu/ES/humsoc/Galileo/Things/ptolemaic_system.html
  The Ptolemaic System
- http://www.census.gov/ipc/www/wp96.html
  World Population Profile: 1996 is the latest published compendium and analysis of data on population, fertility, mortality, contraceptive use and related demographic topics by the U.S. Bureau of the Census.
- http://www.cdc.gov/nchstp/hiv_aids/stats/hasrlink.htm
  The HIV/AIDS Surveillance Report contains tabular and graphic information about U.S. AIDS and HIV case reports.
- http://usda.mannlib.cornell.edu/usda/usda.html
  The USDA Economics and Statistics System contains nearly 300 reports and data sets from the economics agencies of the U.S. Department of Agriculture.
- http://www.eia.doe.gov/price.html
  Links to Energy Price Data and Forecasts
- http://www.runet.edu/~libr-web/other/statinfo.html
  The following resources will provide starting points for finding a wide variety of statistics, scientific data, and social science data.

6 Conclusion

The course is being taught for the first time during the Spring 1998 quarter. A web site containing the Mathematica packages and notebooks, as well as a description of the experiences from the class will be developed during the summer 1998. For additional information or the URL of the web site, please contact the author at sheubac@calstatela.edu.

References

[1] *Mathematica Version 3.0*, Wolfram Research Inc., 1996.

[2] *Maple V Release 5*, Waterloo Maple Software, 1997

[3] Hubbard, John H. and Beverly H. West, *MacMath: A Dynamical Systems Software Package for the Macintosh*, Springer Verlag New York, 1992.

[4] *Statistical Abstracts of the United States 1997*, http://www.census.gov/statab/www/

[5] Poundstone, William, *Prisoner's Dilemma*, Anchor Books, Doubleday, 1992