AN INNOVATIVE APPROACH TO MODELING AT THE FRESHMAN/SOPHOMORE LEVEL

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1 Introduction

This paper presents an overview over a new modeling course designed with a Course and Curriculum Development grant from the National Science Foundation[1]. The unique feature of this course is its intended audience: Non-science majors, especially students who take only one mathematics course during their undergraduate program. For most of these students, mathematics applications consist of balancing checkbooks and computing percentages. This course will give them a better appreciation of mathematics. Based on applications from across the disciplines, the course focuses on model development, test of model against data and interpretation of results. Both discrete and continuous models are used for deriving quantitative and qualitative interpretations of long-term behavior and sensitivity to model parameters. Advanced computations are performed by the computer algebra system Mathematica using an intuitive point-and-click interface and special packages developed by the author. This customized interface frees the student from the usual beginner's frustrations of learning Mathematica, and assists in interactively exploring "What if?" scenarios. The course will provide these students with an understanding of the modeling process, in particular the fact that models need to be validated by data, and that their predictions may be limited. Students will be able to critically evaluate conflicting information on topics such as Global Warming and to put model predictions into the proper context.

2 Incorporation of Technology

Use of technology, whether computers or graphing calculators, introduces an additional level of difficulty for students in any course. To minimize the learning curve and to alleviate the usual beginner's frustrations with Mathematica, the author has developed a set of packages with functions specifically designed for the modeling course, as well as an intuitive point-and-click interface based on the palette feature of Mathematica 3.0.

For each major section of the course, a package with functions relating to the particular topic has been developed. These functions are pasted into the notebook by clicking on a

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button in the corresponding palette, thus avoiding frustrations from typos or incorrect use of parentheses and brackets. These palettes can be accessed from within any notebook via the File menu. In tandem with each palette, a help file explaining the different palette functions is available on-line. Each help file gives an introduction on how it is to be used, and has hyperlink buttons for each palette function described within. Clicking on a button links the student to a full description on how to use the particular function, followed by a specific example.

3 Course Content

The course starts by describing the sequence of the models for the solar system proposed over the centuries. This illustrates the iterative nature of the modeling process, where comparison of model predictions and data may lead to the realization that the model needs to be revised. The remainder of the course is divided into three principal parts: Part I defines the concept of a function and leads to a model derivation by means of a least squares fit to a set of data. Part II derives discrete models (both single and multiple equations) from assumptions, while part III focuses on continuous models.

3.1 Functions and Least Squares Fit

Using the input-output paradigm, the concept of a function is defined, and the various representations (list of values, graph, or formula) for functions are presented. Polynomials, exponential, logistic and sine functions are introduced. Students explore characteristics such as constant first or second unit differences as well as features of the shape of the graph of the different types of functions. These properties are then used to decide which type of function should be fitted to given real-world data from sources such as the Statistical Abstracts. The method of least squares is illustrated graphically to give students an understanding on how the fitted function is derived. The package *DataFit* contains special functions to compute first and second unit differences as well as unit ratios to test for either linear, quadratic or exponential functions. In addition, there are package functions to produce a least squares fit for all the function types discussed. The result of such a fit is a graph of the data together with the fitted function, as well as the functional expression of the least squares fit function.

3.2 Discrete Models

Discrete models, for both single equations and systems of equations, are derived from assumptions using the paradigm

new = old + change.

Examples such as paying off debt on a credit card are used to define and discuss equilibrium values, stability and long-term behavior in a concrete context. Using the *DDS1* package, which contains the functions for discrete models of one equation,

students can display model values in numerical form or through a graph. Animations of Cobb-Webb graphs demonstrate the dynamic nature of the model and are used to visualize the stability of the equilibrium value(s). Furthermore, the questions asked in the context of the problems motivate why deriving a general solution is desirable.

From the example of the credit card with known underlying dynamics (monthly interest rate and payment amount), the students progress to applications that illustrate the iterative nature of the modeling process: Deriving a model, solving it (with the help of Mathematica), comparing it to data, and, if necessary, to refine it. A prime example is population data. The simplest model assumes that the increase in population is proportional to the current population. Using partial data on California's population, an exponential model is derived. However, when comparing this model to the full data set, it becomes obvious that the model needs to be modified, leading to a logistic model. The chapter on single equation models ends with a look at the chaos equation $x(n+1) = x(n)(1- x(n))$ to illustrate the sensitivity of the model behavior to the model parameters.

The chapter on discrete dynamical systems looks at two or more quantities of interest whose values depend on each other. Examples are predator-prey systems and infectious diseases. Starting from the set of equations, matrix notation is introduced as a way to capture the essence of the system. The *DDSn* package (for discrete dynamical systems of *n* equations) includes functions such as LiveVecs, which creates an animation of pairs of input/output vectors (including the Eigenvectors) of a 2-by-2 matrix. This animation allows students to discover the special vectors for which the output vector is a multiple of the input vector.

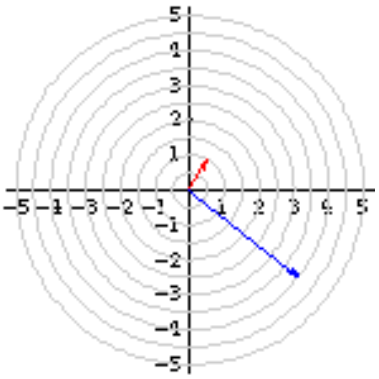


Figure 1: Graphical representation of input and output vector

At this point, students compute several iterations of the system values and are guided towards the realization that the solution can be expressed in terms of the Eigenvectors and Eigenvalues. After this illustrative example, students are now using a palette function to display the general solution to the system in terms of the Eigenvalues and Eigenvectors. From this form, the long-term behavior of the system can be determined.

Additional examples discussed in the section on discrete systems include population models that incorporate age structure, simple probabilistic models and an example of game theory. One of the probabilistic models deals with genetics. For this example, students start with a hands-on activity to understand how randomness can be simulated. Later on, simulations on a WWW page are used to illustrate how much more likely the loss of genetic material (genetic drift) becomes if a population is too small.

Game theory is used to explain why animals cooperate with one another to their mutual benefit, even though a single animal could do better if it would take advantage of the cooperation of the group. Students will play a sequence of prisoner's dilemma games against the computer, where the choice is to cooperate or to defect at each turn. The goal is to find a strategy that results in as high a payoff as possible. The computer employs tit-for-tat, that is, it uses the student's last move as its next move. This leads to a discussion of the overall performance of a strategy and also to detecting the computer's strategy. Based on this simulation, students explore what happens when animals with different "genetic" strategies interact with each other and which "genetic" strategy will be surviving.

3.3 Continuous Models

Deriving model equations in the continuous setting involves the rate of change of a quantity. Using the example of average and instantaneous velocity, the interpretation of the derivative as rate of change is introduced. Side by side with the numerical derivation, the graphical interpretation of the derivative as the limit of secant slopes is developed and visualized through an animation. The two notions of the derivative are combined for the qualitative analysis of a differential equation. Using the slope field, students graphically derive the model solution and discuss the sensitivity of the long-term behavior to initial conditions. Examples used are exponential decay and the continuous version of systems like the predator-prey model. All models are restricted to one or two variables, as these are the only ones that can be displayed visually using vector fields.

4 Pedagogy

A major concern in any course that relies to a great amount on technology is the creation of a black-box approach, where the computer is the source of all answers blindly accepted by the students. To avoid this problem, students are performing computations initially using paper and pencil (whenever suitable) to strengthen their conceptual understanding before using the corresponding palette functions. In addition, students are required to make predictions about the model results before solving the model using Mathematica. Model results are discussed and interpreted in the context of the problem both orally and in writing.

Student assignments consist of exercises reinforcing new definitions or methods, as well as open-ended questions that require both computations and writing. These exercises extend models presented in class by either asking additional questions based on the model or by introducing variations to the model. A final class project guides students through the complete modeling process: Formulating a model, testing it against data, discussing possible modifications and justifying their final choice for the model. These projects are presented to the class either orally or as a poster session.

5 Student Responses

The course was first taught during the Spring 1998 quarter at CSLA, and is currently being taught for the second time. Generally, student reaction was positive. Here are two student comments from the course evaluation:

> *"This class has been a great ego booster in regard to my math inadequacies. I usually have no concept of math and for once I have been understanding it"*
>
> *"Most of my experiences with math courses have always been somewhat uninteresting and frustrating. Mathematica however, is a new approach that is still very challenging but adds a new element - it is interesting and you really get to see how math works outside the classroom. I'll recommend the course to other students"*

6 Internet Resources

The Internet is used for simulations, background information, and as a rich source of data that can be used for testing the various models. Below is a short list of interesting WWW sites for use in this type of course.

- http://www.eia.doe.gov/price.html: Links to Energy Price Data and Forecasts
- http://www.census.gov/ipc/www/wp96.html: World Population Profile 1996
- http://www.cdc.gov/nchstp/hiv_aids/stats/hasrlink.htm: HIV/AIDS Data
- http://usda.mannlib.cornell.edu/usda/usda.html: Economics Data
- http://www.fedstats.gov/index20.html: Links to all Federal Agencies.
- http://www.census.gov/statab/www/: Statistical Abstract of the United States

7 Course Material

The lecture notes, packages, palettes, and introductory Mathematica lessons are posted on the WWW (URL: www.cinenet.net/~silvi)*;* this site can also be accessed via link from the author's home page (URL: www.calstatela.edu/faculty/sheubac/sheubac.htm). Please send any feedback on the lecture notes, packages, or the Mathematica lessons to the author at sheubac@calstatela.edu.