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Reply to Armour-Garb

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Hermeneutic non-assertivism is a thesis that mathematical fictionalists might want to endorse in responding to a recent objection due to John Burgess. Brad Armour-Garb has argued that hermeneutic non-assertivism is false. A response is given here to Armour-Garb's argument.

Let *mathematical fictionalism* be the view that (a) our mathematical theories do purport to be about abstract objects, as platonists claim, but (b) there are no such things as abstract objects, and so (c) our mathematical theories are not true. John Burgess [2004] argues against fictionalism by arguing for the following two theses:

(i) Fictionalists have to endorse either hermeneutic or revolutionary fictionalism (where *hermeneutic fictionalism* is the view that typical mathematicians intend their theories to be taken as fictions, and *revolutionary fictionalism* is the view that when typical mathematicians utter ordinary mathematical sentences, what they say is false, or at least not true).
(ii) Revolutionary and hermeneutic fictionalism are both untenable.

In my [2009], I argue that (i) and (ii) are both false. My primary response to Burgess is that (ii) is false. I admit that hermeneutic fictionalism is untenable, but I argue that the sort of revolutionism that fictionalists would be committed to here, if indeed they were committed to revolutionism, is perfectly acceptable.[1]

Again, that's my main response to Burgess. But I also argue that (i) is false. I argue that fictionalists can avoid both hermeneutic and revolutionary fictionalism by endorsing the following (admittedly controversial) empirical hypothesis:

> *Hermeneutic Non-Assertivism* (HNA): When typical mathematicians utter sentences like 'Every number has a successor' and '4 is even', they should not be interpreted as saying what these sentences say,

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[1] The strategy of defending revolutionary fictionalism has also been pursued by Mary Leng [2004] and Chris Daly [2006].

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and indeed, they should not be interpreted as saying anything, *i.e.*, as asserting propositions at all.

I admit that this is wildly controversial, and I do not offer any positive argument for it in my paper. But I claim that it is not obviously false, and I defend it against a certain objection. Thus, my stance here is that while fictionalists do not need HNA in order to respond to Burgess (or for any other reason), it is a view that they might want to consider.

In his [forthcoming], Brad Armour-Garb argues that HNA is false. His argument proceeds roughly as follows:

(1) Mathematicians have the concept *number*, and they accept various number sentences, *e.g.*, '3 is prime', '$2 + 2 = 4$', and so on. Therefore,
(2) Mathematicians understand their mathematical sentences. But
(3) If mathematicians understand these sentences, then the best account of what they are doing when they utter them is that they are making assertions. Thus,
(4) When mathematicians utter mathematical sentences, they are making assertions, and so HNA is false.

Armour-Garb spends most of his time arguing for (2). But it seems to me that HNA-ists should just accept (2). What they should say, it seems to me, is that (3) is false. And Armour-Garb offers no argument at all for (3).

To put the point a bit more slowly, I think HNA-ists should respond to Armour-Garb by saying that (a) mathematicians do understand mathematical sentences (HNA-ists do not need to say that understanding mathematical sentences requires knowing the metaphysical nature of mathematical objects), and (b) the claim that mathematicians understand mathematical sentences does not undermine the HNA-ist hypothesis that when typical mathematicians utter mathematical sentences, they do not make assertions.

It is worth noting that one might distinguish multiple kinds of asserting, and HNA-ists can allow that there may be various senses in which mathematicians do make assertions. What HNA-ists need to maintain is that when typical mathematicians utter ordinary mathematical sentences, they do not make what might be called *W-assertions–i.e.*, they do not assert any *propositions*. Thus, in response to Armour-Garb, what HNA-ists should say is that the fact that mathematicians understand mathematical sentences is not good evidence for the claim that typical mathematicians make W-assertions with their mathematical utterances.

You might wonder what HNA-ists think mathematicians *are* doing when they utter mathematical sentences, if they are not asserting the propositions expressed by those sentences. I have not taken a stand on what HNA-ists should say about this, but it seems to me that the most plausible

versions of HNA involve the idea that when typical mathematicians utter mathematical sentences, they are doing something that differs from asserting (or W-asserting) in a pretty subtle way, so that the difference between W-asserting and this other kind of speech act is not obvious. (And, again, it is important to remember that I am not *endorsing* HNA; I am just putting it out there as a possible view.)

Similar remarks apply to another of Armour-Garb's arguments. I pointed out in my [2009] that opponents of HNA might argue that when mathematicians utter mathematical sentences, we should take them as making assertions because they think their sentences are *true*. I responded by claiming that (a) it is not clear that the sort of truth that mathematicians are working with is equivalent to the sort of truth that fictionalists are working with when they deny that our mathematical sentences are true (the latter sort of truth, which we can call *W-truth,* is a sort of truth that (typically) involves accurate description of actually existing objects), and (b) if the sort of truth that mathematicians are working with is not W-truth, then it's not clear that we have good reason here to think that mathematicians are making assertions (and let me clarify this now by saying that by 'assertions', I mean *W-assertions*).

Armour-Garb responds to this by pointing out (correctly, I think) that one does not need to know what truth *is* in order to be a competent user of 'true'. He also accuses me of maintaining that mathematicians are not working with the "right" kind of truth. But I would not say that if mathematicians are working with a *different* notion of truth, then it is not the *right* notion. And I also would not deny Armour-Garb's claim about what is needed for being a competent user of 'true'. I simply want to say the following: (1) I admit that if mathematicians think their utterances are W-true, then it is best to interpret them as making W-assertions; but it is not clear that mathematicians do think their utterances are W-true because it is not clear that the notion of truth they're working with is W-truth; and if mathematicians are indeed working with a different notion of truth, distinct from W-truth, then it is not clear that the fact that they think their utterances are true is reason to conclude that they are making W-assertions. And (2) all of this is perfectly consistent with admitting that (a) our mathematical theories are true in some sense of the term (and hence that mathematicians are not necessarily mistaken in thinking that their theories are true) and (b) when mathematicians utter mathematical sentences, they are making assertions of some kind. HNA-fictionalists can admit that (a) and (b) are both true. For when they say that our mathematical theories are not true, what they mean is that they are not *W-true*, *i.e.*, they do not have the sort of truth that (typically) involves accurate description of actually existing objects; and when they say that typical mathematicians do not make assertions with their mathematical utterances, what they mean is that they do not make *W-assertions*.

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