Mark Balaguer

Why Mathematical Fictionalism isn't Psychologistic

Abstract: *This paper provides comments on Susan Schneider's paper 'Does the Mathematical Nature of Physics Undermine Physicalism?'. In particular, it argues that, in contrast with what Schneider suggests, mathematical fictionalism is not a psychologistic view in any interesting sense.*

The purpose of this paper is to provide a few comments on Susan Schneider's paper 'Does the Mathematical Nature of Physics Undermine Physicalism?' Before I start, I'd like to say that I think this is a really good paper — it's both interesting and original. I disagree with Schneider's central argument, but of course, that's normal in philosophy.

I have two main worries about Schneider's argument, but I'll discuss only one of them here. The one I won't discuss has to do with the notion of an individuating condition. *Contra* Schneider, I think that when we give such conditions, we're just doing conceptual analysis, and, on my view, conceptual analysis is ultimately just empirical semantics. So when we give the individuating conditions for Fs, we're not saying anything about non-semantic reality; we're just making a trivial point about what 'F' means in ordinary English. Schneider says a few words about views of this kind in Section 7; if I had more space, I would discuss this issue here, but unfortunately, I don't.

What I want to discuss instead is what Schneider says about mathematical fictionalism. She seems to think that fictionalism is a psychol-

Correspondence:
Mark Balaguer, Department of Philosophy, California State University, Los Angeles, CA, USA. *Email: mbalagu@exchange.calstatela.edu*

Journal of Consciousness Studies, **24**, No. 9–10, 2017, pp. 103–11

ogistic view (e.g. she says that it 'explains mathematical discourse in terms of mental phenomena'), and she also thinks that if mind–brain physicalists endorse fictionalism then they'll be committed to saying that there are mental phenomena in what she calls the physical base (or that there are things in the physical base that are 'individuated by mental entities').

I think this stance involves a mischaracterization of the fictionalistic view of mathematics. I don't think fictionalism is psychologistic in any interesting sense — indeed, I think it's every bit as anti-psychologistic as mathematical *Platonism* is. If I'm right about this, then it undermines Schneider's claim that mind–brain physicalists can't endorse mathematical fictionalism. Indeed, as far as I can see, there's no reason at all that physicalists can't endorse fictionalism.

In order to argue that fictionalism isn't psychologistic, I need to say a bit to develop the view. For starters, we can define the view as follows:

> *Mathematical Fictionalism*: (a) The Platonist semantics for mathematics is correct — that is, our mathematical sentences and theories are about (or at least purport to be about) abstract objects; but (b) there are no such things as abstract objects; and so (c) our mathematical sentences and theories are not true. (Thus, for example, the sentence '3 is prime' is not literally true. It's not true for the same reason that, say, 'Santa Claus is jolly' isn't true — because just as there is no such thing as Santa Claus, so too there is no such thing as the number 3.)

But while fictionalists think that sentences like '3 is prime' are literally untrue, they obviously can't say that these sentences are entirely worthless — there's obviously *something* right about these sentences, and fictionalists need to account for this. They need to recover a sense in which sentences like '3 is prime' are *right*, or *correct*, whereas sentences like '4 is prime' are not. Hartry Field (1989) has argued that fictionalists can do this by claiming that '3 is prime' is *true in the story of mathematics*, whereas '4 is prime' is not. This, I think, is a good start, but fictionalists need to say what this means. Field's view (see, for example, his 1998) is that (a) the story of mathematics consists in the axiom systems that are currently accepted in the various branches of mathematics, and (b) truth in this story amounts essentially to following from these axioms. Elsewhere (2009), I've argued that this is problematic and that fictionalists should employ the following definitions instead: (a) the so-called

story of mathematics is just the claim that Platonism is true (or more precisely, it's the claim that *plenitudinous* Platonism is true — more on this in a bit); and (b) a sentence is *true in the story of mathematics* iff (roughly) it would have been true if Platonism had been true — or, more precisely, iff it would have been true if there had actually existed a (plenitudinous) realm of abstract mathematical objects. (The reason the second formulation is a better characterization of what fictionalists believe (when they say that sentences like '3 is prime' are true in the story of mathematics) is that the first formulation involves a claim about *Platonism*, and Platonism is presumably an abstract object (in particular, a proposition), and fictionalists don't believe in abstract objects.)

So fictionalists claim that while '3 is prime' isn't strictly true, there's still an important sense in which it's *correct* because it's true in the story of mathematics. Moreover, according to fictionalists, the sort of correctness that's at work here — *fictionalistic correctness*, we might call it — is perfectly objective and factual. To appreciate this, notice that according to fictionalists, the reason that '3 is prime' is true in the story of mathematics is that the following nearby sentence is literally and objectively true: 'If there had actually been a plenitudinous realm of abstract mathematical objects, then it would have been the case that 3 was prime.' Finally, it's also important to note that the sentences that come out true in the story of mathematics on the fictionalist view are the very same sentences that come out true on the Platonist view; in other words, the two views divide the mathematical sentences into the 'good' ones and the 'bad' ones in an extensionally equivalent way.[1]

In any event, given the above characterization of fictionalism, it might be surprising that Schneider thinks of this view as being somehow psychologistic because, at first blush, it doesn't seem to be

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1 You might think that this is all unhelpful because counterfactuals of the above kind commit to the existence of abstract objects or possible worlds. But I think this is wrong. I can't argue the point here, but I think that counterfactuals like this can be true even if there are no such things as abstract objects or possible worlds. Roughly speaking, this is because the antecedents of these counterfactuals *entail* the consequents; but I can't say any more about this here. Another worry you might have about all of this is that if fictionalists say that mathematical objects *couldn't* exist, then the counterfactuals they're committed to here will be counter*possibles*. But I don't think this is a problem because I think that (a) counterpossibles can be straightforwardly (and non-vacuously) true, and (b) anti-platonists can (and should) endorse contingentism about abstract objects. But, again, I can't get into any of this here.

psychologistic at all. Fictionalists think that our mathematical theories are about (or purport to be about) abstract objects, not mental objects. Moreover, the way in which fictionalists think that our mathematical theories are correct doesn't have anything to do with any mental phenomena either; the reason that sentences like '3 is prime' are fictionalistically correct on this view is that certain counterfactuals are literally true, and these counterfactuals aren't about any mental phenomena — they're about what would have been the case if Platonism had been true.

So why does Schneider think of fictionalism as a psychologistic view? Well, it's not obvious. One thing she might have in mind here (although, frankly, I doubt it) is that fictionalists think that mathematics is a *human creation*. Now, I suppose it's true that fictionalists would say that mathematics is a human creation, but this doesn't make their view psychologistic. After all, Platonists would presumably say that mathematics is a human creation as well — they would say that *we* came up with these theories of abstract objects — but this doesn't make their view psychologistic. Now, you might have thought fictionalists think that mathematics is a human creation in a second sense, for you might have thought that fictionalists think that humans have created not just our mathematical theories but the objects that these theories are about — i.e. things like numbers and sets and functions. But fictionalists (of the kind I'm describing here) *don't* think this. They don't believe in mathematical objects at all. They think that there are no such things as mathematical objects, so they obviously don't think that we've created such objects.

Are there any other reasons Schneider might have for thinking of fictionalism as a psychologistic view? I think there is; for one might think that on the fictionalist view facts about our thoughts sneak into the correctness conditions of our mathematical sentences and theories.

I think this is a mistake. I don't know if it's a mistake that Schneider is making, but it's a mistake that people often make about mathematical fictionalism. In any event, in what follows, I want to explain (a) why one might think this about the fictionalist view, and (b) why it's not true. To see why one might think that fictionalism is psychologistic in this way, consider the following speech one might make:

> Fictionalists think that mathematical correctness comes down to truth in a certain mathematical story. But there are lots of different mathematical stories, and different sentences come out true in different stories. For instance, according to one story, the largest number is 100: in this story, '50 + 51 = 101' isn't true. This story might not be very useful to us (or very interesting, or aesthetically pleasing), but it's hard to see how fictionalists could say that it's any worse than the standard arithmetical story in terms of truth. According to fictionalists, both of these stories are untrue fictions. The only difference is that one of them is *our* fiction. Given this, it should be clear that if fictionalists want to obtain the result that the sentences that are true in the story of mathematics are precisely the sentences that we ordinarily think of as true, then *not just any story will do*. Fictionalists need to say that what matters is truth in *our* story of mathematics. They can do this in the above counterfactual way if they want to — i.e. they can say that the mathematical sentences that count as fictionalistically correct are the ones that would have been true if abstract mathematical objects had existed. But, again, not just *any* bunch of (fictitious) abstract objects will do here. We're not interested in what would have been true if the numbers 1–100 (and no others) existed. Fictionalists have to say that what we're interested in is what would have been true if there had been abstract objects *of the kinds that we have mind when we do mathematics*. And what these objects *are* — or what they would have been like — is completely determined by facts about our heads. In particular, it's determined by *mental* facts about what we have in mind when we do mathematics.

I now want to explain what's wrong with this speech. The first point I want to make here is that according to the fictionalist view I'm describing here, the relevant story of mathematics is the claim that *plenitudinous* Platonism is true; i.e. it's the claim that all the mathematical objects that *could* exist actually *do* exist. So on this view, the story of mathematics gives us *all* the objects that we might want to talk about; e.g. it gives us the natural numbers, and it *also* gives us structures in which the largest number is 100.

Second, while I agree that on the fictionalist view, facts about our heads are relevant to determining which sentences count as true in the story of mathematics, this does *not* make the view psychologistic. To bring this point out, I want to explain why (a) platonists need to make an exactly analogous claim (in particular, they need to say that facts about our heads are relevant to determining which mathematical sentences count as *true*), and (b) we all need to make an analogous claim about ordinary sentences about ordinary physical objects (i.e. we need to say that facts about our heads are relevant to determining which of these sentences are true). To appreciate these points, I need to say a bit more about the Platonist view of mathematics.

There are strong reasons to think that Platonists should endorse plenitudinous Platonism. I have argued this point at length elsewhere (1998). I can't get into this here, but the most important point is

(roughly) that if Platonists go for a non-plenitudinous view (i.e. if they say that certain possible kinds of abstract objects are uninstantiated), then (a) they'll be committed to certain untenable kinds of metaphysical arbitrariness and anthropocentrism about the nature of the Platonic mathematical realm, and (b) they won't be able to construct an acceptable epistemology — i.e. they won't be able to explain how human beings could acquire knowledge of abstract objects.

Given that Platonists have to (and should want to) endorse plenitudinous Platonism, they're going to say that every internally consistent (purely) mathematical theory accurately characterizes a collection of abstract objects, or a part of the mathematical realm. Thus, for example, they're going to allow that there are some abstract mathematical structures that are accurately characterized by sentences like

(100) 100 is the largest number.

Now, Platonists obviously don't want to say that this sentence is *true*; so they need to tell a story about which consistent mathematical sentences and theories count as true; and this is a bit tricky because, again, Platonists are committed to saying that *all* of these sentences and theories accurately describe parts of the mathematical realm. I think it can be argued that what Platonists should say here is (roughly) that a mathematical sentence S is *true* iff it's true in the *intended* structure — i.e. the structure that *we have in mind* in the given branch of mathematics. (More precisely, they should say that S is *true* iff it's true in *all* the parts of the mathematical realm that count as intended in the given branch of mathematics; but I won't worry about this complication here.) Given this, Platonists can say that while sentences like (100) accurately characterize real mathematical structures, they're *not true* because they're not true in the intended structures — i.e. the structures that we have in mind when we do mathematics.

So on the Platonist view, facts about our heads are relevant to determining which mathematical sentences count as true. And there's nothing odd about this. We all need to say the same thing about just about all of our sentences. Consider, for example, 'Snow is white'. This sentence is true partly because it means in ordinary English that snow is white, and partly because snow is white. So facts about our heads (in particular, about what we mean by our words) are relevant to determining which of our sentences are true. And the same goes for sentences couched in the language of physics — e.g. 'Electrons are negatively charged'.

The above point about what Platonists should say about mathematical truth is exactly analogous to the point just made about what we should all say about the truth of sentences like 'Snow is white' and 'Electrons are negatively charged'. The reason that facts about our heads are relevant to determining which mathematical sentences are true, according to Platonists, is that those facts are relevant to determining what our mathematical sentences are *about*. More precisely, facts about our heads are relevant to determining which parts of the abstract mathematical realm we're talking about when we utter our mathematical sentences. But once the facts about our heads have done this work, they don't do any further work in determining whether our mathematical sentences are true. And, again, this is exactly analogous to what we should all say about ordinary sentences like 'Snow is white'.

The point I want to make now is that this is exactly analogous to what fictionalists say about the role that facts about our heads play in determining which sentences count as true in the story of mathematics. According to the sort of fictionalism described above, a mathematical sentence is true in the story of mathematics iff (roughly) it would have been true if Platonism had been true. But now that we know a bit more about the Platonist view, we can see that this is equivalent to the following:

> A mathematical sentence S is *true in the story of mathematics* iff the following is true: if there had actually existed a plenitudinous realm of abstract mathematical objects, then S would have been true in all the structures that count as intended in the given branch of mathematics — i.e. all the structures that fit with what we have in mind in the given branch of mathematics, or some such thing.

On this view, facts about our heads are relevant to determining which mathematical sentences count as correct, or true in the story of mathematics. But this is exactly analogous to the role that facts about our heads play in determining which mathematical sentences count as true on the platonist view, and this is in turn analogous to the role that facts about our heads play in determining which ordinary sentences about the physical world count as true. In particular, fictionalists can say that the relevant facts about our heads are relevant only to determining what our sentences are about — i.e. what they're saying about the world. Beyond that, facts about our heads play no role, on the

fictionalist view, in determining which mathematical sentences count as correct, or true in the story of mathematics.

I take all of this to show that mathematical fictionalism is not a psychologistic view of mathematics. At any rate, it's no more psychologistic than Platonism is, and it's no more psychologistic than standard scientific realism is or standard common-sense realism about the physical world is. On all of these views, facts about our heads are relevant to determining which of our sentences are true (or fictionalistically correct), but this is only because they're relevant to determining what our sentences are about — i.e. what they're *saying*. But once this has been determined, facts about our heads are not relevant, on any of these views, to determining whether what we're saying with our sentences is true (or fictionalistically correct). On the fictionalist view in particular, facts about our heads are relevant to determining what exactly we're saying when we utter sentences like '3 is prime'. But once this has been determined, facts about our heads play no role whatsoever in determining whether sentences like this are correct, or true in the story of mathematics. According to fictionalists, this is determined by counterfactual facts about what reality would have been like if there had existed a plenitudinous mathematical realm. And on the fictionalist view, facts of this kind are objective, non-psychological facts; e.g. it's an objective, non-psychological fact that if there had been a plenitudinous realm of abstract mathematical objects then it would have been the case that 3 was prime.

Perhaps we can sum all of this up by saying that on the fictionalist view, facts about our heads *aren't part of what we're talking about* when we utter mathematical sentences; they're relevant only to *determining* what we're talking about.

The upshot of all of this, *vis-à-vis* Schneider's paper, is that physicalists about the mind–brain can endorse fictionalism about mathematics without committing themselves to the claim that there are mental objects or phenomena in the physical base. Since fictionalists think that mental phenomena aren't part of what we're talking about in mathematics, they can say that mental phenomena aren't part of the physical base. And so I don't think there's any reason why mind–brain physicalists can't endorse mathematical fictionalism.

Of course, if physicalists endorse mathematical fictionalism then they won't be able to say that our physical theories are literally true. They'll have to say that these theories are (at best) *for-all-practical-purposes true*. More specifically, they'll have to say that these theories

are (at best) fictionalistically correct — i.e. such that they would have been true if Platonism had been true. Now, you might think this is a problem, and if you do, you're not alone because this is just the Quine-Putnam indispensability problem. I have argued elsewhere (1998) that fictionalists can solve this problem; in other words, I've argued that there's nothing wrong with claiming that our best empirical theories are (at best) true in the story of mathematics. But, unfortunately, I can't get into this here.

References

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